The role of mathematical FICTION in the learning of mathematics in primary school

JANICE PADULA

provides us with some wonderful ideas for integrating mathematics and literature into the classroom.

This article classifies and describes a selection of mathematical fiction. It also provides some practical activities teachers or parents can use to help make the mathematics more explicit and engaging for their children. Not many people, apart from primary teachers, are aware of mathematical fiction or mathematical picture storybooks, although the study of such books is not a completely new phenomenon (Cullinan 1971; Griffiths & Clyne, 1991; Padula & Stacey, 1990; Reeves, 1984; Smith & Wendelin, 1981; Straine, 1969). The history of the mathematical storybook probably began as long ago as 1865 with *Alice's Adventures in Wonderland* by Lewis Carroll, a pseudonym of Charles Dodgson, mathematician, or earlier still with parts of Swift's *Gulliver's Travels* in 1726 (Swift, 1995).

The mathematics that can be learnt from mathematical fiction varies enormously, as is noted by Anderson, Anderson & Shapiro (2000). They cite Golden and Gerber (1990, p. 205): 'A picture book narrative… has the potential of generating multiple interpretations depending upon what the symbol offers, what the reader brings and how the text is mediated by participants in a social context.' Research about the effects of reading picture storybooks to a child by a caring adult in the home is not extensive. The research about the effects of the interaction between a child and caring adult through the reading of mathematical storybooks is sparse indeed.

Donaldson's (1978) notion of contextualised learning informs Anderson, Anderson and Shapiro's research to the extent that they argue that storybook reading is a site for contextualised mathematical learning. Contextualised learning will aid the understanding of mathematical language, a symbol system, and mathematical concepts.

There are (at least) ten different kinds of mathematical picture storybooks.

Ideas about arithmetic

This category includes books about counting: seriation, (e.g. 8 is the number between 7 and 9); cardinal (one, two, three) or ordinal (first, second, third, etc.) number. It also includes books containing simple addition, subtraction, multiplication and division facts.

Many children's mathematical storybooks can be read on two levels. Sometimes the two levels may consist of a child's and an adult's level, or the levels may consist of a younger and an older child's view. The mathematical and scientific facts about the metamorphosis of the butterfly may not be immediately apparent to very young children being read *The Very Hungry Caterpillar* (Carle, 1970). However, they will most certainly become involved with simple counting activities using both cardinal and ordinal numbers, through their visual, auditory and tactile senses when they see the beautiful illustrations, hear the words and feel the holes in the pages. No wonder Eric Carle's book has become such a popular classic and is still in print more than thirty years later!

In Anno's *Magic Seeds* (Anno, 1995) the former primary teacher Matsumasa Anno not only uses arithmetic within the story but uses it to move his narrative forward. Jack, a poor and hungry man is given a 'magic' seed by an old man who tells him to bake one seed and eat it, then plant the remaining one that will yield two 'magic' seeds. Aided by some beautifully simple illustrations the young reader is encouraged to count ordinal and cardinal numbers, and to multiply and subtract.

Another good example is *Six Foolish Fishermen* (Elkin, 1968). The six foolish fishermen catch many fish, but they are crying. Why? Each brother, checking to see that all are safe after a day's fishing counts the five brothers he can see but forgets to count himself! A small boy solves the problem. This tale is greatly appreciated by six year olds who have just mastered the skill themselves. It gives lots of counting practice and aids understanding of conservation — for the set is rearranged for the same result. Children could be encouraged to dramatise the story or make fish with coloured paper, paper-clip mouths and a magnet fishing line, as Reeves (1984) suggests. Graph the results of the fish 'caught' on a pictogram or block graph with a written explanation of the results.

Relational terms

Books that are rich in relational terms which are necessary to teach concepts of space: 'around', 'in', 'under', 'above'; number: 'more', 'less', 'most', 'few', 'fewer'; time: 'before', 'after', 'day', 'week', capacity: 'full', 'empty', 'hold', and so on. These are especially important for learning the language of instruction by very young children and for learning basic mathematical concepts. Two books rich in relational terms are *Rosie's Walk* by Pat Hutchins (1968) and the Berenstains' (1971) book, *Bears in the Night*, both of which teach concepts of space in a very amusing way.

In *Rosie's Walk*, Rosie the hen journeys around the farmyard pursued by a designing fox. Rosie returns home safely, but the fox ends up being chased by a swarm of bees. The vocabulary of position is supplied by the text. Words like 'around', 'between', 'under' mark Rosie's progress and are given meaning in a humorous context. Spatial relations and order can be developed further after reading the story.

Encourage the children to dramatise Rosie's walk or create an obstacle course outdoors. Use the physical education lesson to copy Rosie, go round the tree, over the logs, through the pipe, etc. A series of children's paintings (or collages) of events in the story can be sequenced, displayed and used as an aid to retelling and rehearsing the mathematical vocabulary (Reeves, 1984).

Reinforce important mathematical ideas

Books that reinforce important mathematical ideas, such as the measurement of volume by displacement in Pamela Allen's *Mr Archimedes' Bath* and the relationship between mass and capacity in the same author's *Who Sank the*

Boat? With *Mr Archimedes' Bath*, trial and error leads to an understanding of displacement by the characters and draws on the children's own experiences at bath time. It is also an inspiration for experimenting with measurement of volume by displacement. Similar experiments can be carried out with (toy) animals of varying mass and a toy or home made boat, after a reading of *Who Sank the Boat?* After listening to *Mr Archimedes' Bath* young children could mark the water level on a container, place an object in the water and watch the water level rise. Comparing the volumes of differently shaped containers and ordering the objects according to volume is the next step.

Older students can use calibrated containers and measure the displacement in millilitres. A 10 cm × 10 cm × 10 cm container that is marked to a litre would demonstrate the relationship between 1 mL and 1 cm^3, allowing for the volume of objects to be described in both millilitres and centimetres (Reeves, 1984).

Also for older children, Enzensberger's (1999) book *The Number Devil* (written for 12 year-olds and upwards) covers such topics as number theory, fractal geometry, and famous mathematicians, and moreover shows the connections between topics, something sometimes ignored by the traditional mathematics textbook writers. All this is accomplished within the framework of an imaginative story of a boy who hates the maths he does at school and dreams every night of a clever 'devil' who helps him to understand it.

Suitable for the older (twelve years and up) reader, Guedj's (2000) *The Parrot's Theorem*, is the story of an elderly bookshop owner, Mr Ruche, a single mother with an adopted son and teenage twins, the mystery of a missing theorem, and a talking parrot. This detective story is interspersed with fascinating stories of famous mathematicians, their contributions and their lives. Simple diagrams illustrate the mathematics.

Just as Mr Ruche explains the work of the famous mathematicians, teachers may illustrate stories from the history of mathematics with diagrams, slides, films and experiments such as measuring the shadow of a pyramid (model) with a stake and a rope (or string) just as Thales did when calculating the height of the Great Pyramid of Cheops.

Recipe books

Recipe books, or books that may contain just one single recipe embedded in a story. Recipes are a rich source of mathematical learning for the lower and indeed, the middle- and upper-primary school child. Following a recipe is an excellent opportunity for children to measure mass, volume, capacity and time. Important mathematical ideas can be transmitted such as fractions ('half a cup' etc.), ratio (when one must understand the relationships between the ingredients of a recipe in order to alter them to suit more or fewer people) and the multiplication of fractions which may be demonstrated by a parent or teacher when one substitutes say, two 1/4 cups for a 1/2 cup: 2 × 1/4 = 1/2. (For younger children the fact that 2 × 1/4

cups make 1/2 cup or 4 × 1/4 cup makes one cup may be demonstrated, or discovered by the children themselves with plastic cups, spoons and flour.)

Two good examples of recipe books suitable for young children are *The Grandma Poss Cookbook* (1985) illustrated by Julie Vivas, and, although it does not actually include the pancake recipe, *Mr Wolf's Pancakes* by Jan Fearnley (1999). Mr. Wolf asks each of his neighbours to help him make pancakes. They all refuse so he makes them himself, invites the neighbours to his house and then eats all the pancakes and all the neighbours! The teacher's questions may include: How many neighbours did Mr Wolf ask to help him? How many pancakes did Mr. Wolf make? How many pancakes each would his neighbours have eaten — if Mr. Wolf had not eaten them first?

Of course, pancakes may be made in class with the children measuring mass, volume, capacity and time. Arbitrary units like cup, spoon, pinch, and drop, may be used with younger children and the metric units of kilogram, litre and millilitre with older children.

Sequential thinking

Books and stories that encourage sequential thinking (ordering) in the child. The essence of story grammar is that every story has a beginning, middle and an end; the child's sense of story is an ordering of a kind, as is the sequencing of events in many stories. Books such as Anno's *Counting Book* (1985) may be illustrated with paint, crayons or collages by the children and then arranged in order from zero to 12 — with the clockface on the church steeple showing a different time on each — just as the wonderful Japanese illustrator has drawn his clocks.

All in a Day devised by Mitsumasa Anno (1986) utilises the concept of time zones, showing young children in eight countries on the same page. Children may be playing in the sunshine in Australia while children are sleeping in Italy. Anno has chosen eight accomplished illustrators from different countries to show the actions of these children, from Ron Brooks in Australia to Eric Carle in America.

Logic

Books that involve logic or logical thinking. In *Alice's Adventures in Wonderland* (1999), the conversation between the Hatter and Alice during the mad tea party is a delight. The logic of Alice's day-to-day existence collides joyously with her protagonist's surreal Wonderland logic.

The settings of many stories create worlds with a logic of their own, for example, the Kingdom of Didd in *The 500 Hats of Bartholomew Cubbins* (Dr Seuss, 1965) — a logic that the young child will insist on, once it is established by the writer. Bartholomew must take his hat off in front of the king, but every time he does, there is still a hat on his head! As Cullinan (1971) states: the improbable must occur within the framework of possibility the author has established, especially in fantasy books.

A good example of a book involving logical thinking is Juster's (1989) classic, *The Phantom Tollbooth*, with a tollbooth portal being the entry to a world with rules of its own. Since most of the humour is 'a play on words', older children will probably appreciate the humour more than younger ones.

Anno's *Hat Tricks* (Nozaki & Anno, 1985), illustrated by Anno, introduces readers to binary logic with a character called Shadowchild, a boy called Tom, and a girl called Hannah, and hats of red and white. There is an explanatory note at the back (as is usually the case with Anno's mathematical books), a 'tree' diagram and a flow chart (as in computer programming) for 'parents and other older readers'. Since Shadowchild must determine the colour of the hat on his head by looking at the colour of the hats on Tom's and Hannah's heads, dramatisation of the various situations with the requisite red and white hats, and much debate and discussion, would be most effective with older children.

Patterns

Books that encourage children to look for pattern in books or in nature. Mathematics is the language of pattern recognition. Many of the books about shapes, written for young preschool children are rich in pattern, as are many counting

books. A good example is Anno's *Math Games 2* (1989), section 4, where young children are encouraged to not only develop counting skills, but a sense of what counting is, as drawings of five children, five horses and the like are gradually reduced to circles.

Number patterns are not the only patterns in mathematics and spatial patterns can also be taught through literature. Reeves (1984) describes a spatial pattern of rotational symmetry that can be taught after reading Alan Garner's (1967) novel *The Owl Service* to older children.

> There is a curious pattern on a dinner service that fascinates Allison. She discovers that if she traces around parts of the design then turns the plate a quarter turn, it will make the pattern of an owl. But the mystery begins when Allison finds she cannot stop herself tracing around the shapes on the plates and, by doing so, releasing the owls from their bondage in the dinner service.
> (Reeves, 1984, p. 3)

Reeves suggests using two small mirrors on two sides of a triangle enclosing a simple drawing of an owl. The angles of the mirrors are altered to show the owl drawing repeated (see Figure 1).

Children can draw a small owl on a piece of cardboard, cut it out, retain the cut-out piece or stencil, anchor it with a drawing pin and trace inside the stencil. The cut-out can then be rotated, and traced again and again to create a pattern: four, six or eight times on a circle of coloured paper or a cardboard plate.

Also for older children, Anno's *Mysterious Multiplying Jar* (1999) shows the pattern behind factorials, numbers that are written thus: 4! meaning $1 \times 2 \times 3 \times 4$, and 3! meaning $1 \times 2 \times 3$, an

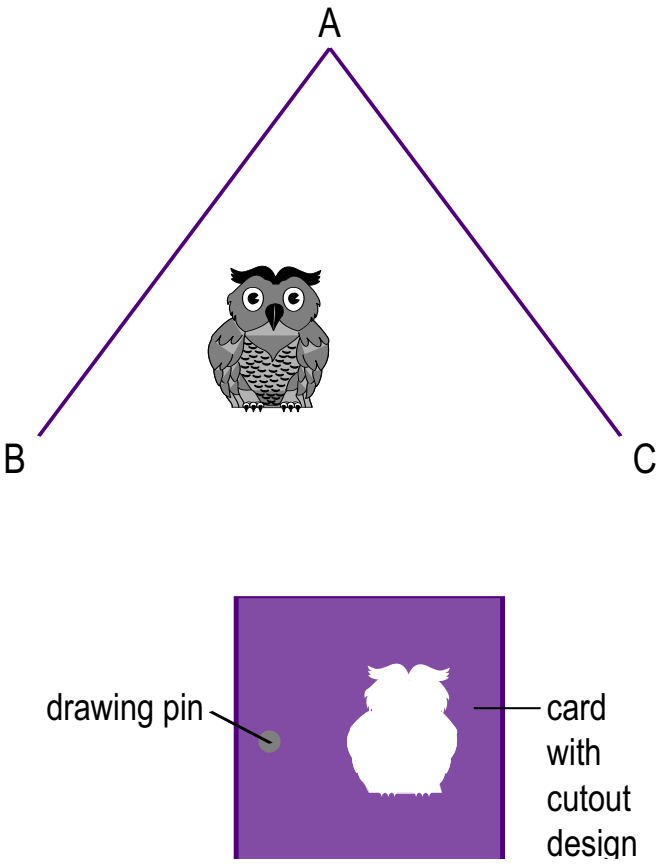

Figure 1. Owl shape and stencil.

important part of the topic 'combinations and permutations', usually taught at high school in Australia.

The book starts with an illustration of a beautiful blue and white jar and when we go inside it (i.e. turn the page) there is water that becomes a wide deep sea, then one island, two countries, three mountains, four walled kingdoms, and so on until the numbers grow so large they are portrayed as small pink dots on a plain white page. This is the best exposition of factorials I have seen and there is a complete explanation of the mathematics by Anno at the back of the book for teachers, parents and older children. Children could be challenged to devise and draw a 'mysterious multiplying jar' of their own.

In *Math-terpieces*, Greg Tang (2003) takes famous works of art, for example, Van Gogh's *Starry Night*, and adds a challenging rhyme asking readers to devise how many different ways they can combine the stars in the famous painting; e.g., 'Find four ways to make a seven.' He has illustrated answers at the back of the book. In this way he addresses a mathematical topic, combinations, in an original way. Children could make the stars, peaches, etc. in a small group and then arrange them in different ways as the teacher writes the children's suggestions on the board. The final count of different combinations could then be seen by the whole class.

Series

Series of books commissioned to cover the entire mathematics curriculum. The American series *MathStart* is written for American levels one to three, with each topic in a separate book. Topics treated include *Patterns*, *Comparing sizes*, *Directions*, and *Recognizing shapes*. Written for younger children, these books seem more pedestrian than imaginative but they may fill a few minutes between packing up and the end of class — especially when the topics have been previously taught. For example, after reading *Slugger's Car Wash* (Murphy, 2002) children can add in dollars and cents the amount Slugger has earned. They could also dramatise the story with real or 'pretend' money.

Picture books

Mathematical picture books which do not contain a strong story line but which contain interesting mathematical/scientific facts. Books in this category include *Mission to Mars* by Branley (2002), which has a foreword written by Neil Armstrong. Working in groups, children will enjoy making the spaceship from waste materials, making a 'space map' showing Mars, the Moon and Earth, and graphing the estimated length of the journey to Mars compared to the journey to the Moon made by Armstrong, and estimating the time both trips would take.

Problem solving

Picture books that contain ideas for problem solving or problems for solving. Two good examples of books that fit into this category are *Maths Curse* (Scieska & Smith, 1998) and *Counting on Frank* (Clement, 1991).

In *Maths Curse*, a young boy starts to think of every event in his day as a maths 'problem' to be solved. The layout, graphics and illustrations are superb, and so is the humour. The topics covered in the book include Fibonacci numbers, logic, fractions and graphing.

Unfortunately a small part of the mathematical language uses imperial measurements — yards, feet, inches, and quarts — but teachers could either leave out one or two problems, or simply change some of the language to suit Australian measuring standards.

In *Counting on Frank*, a young boy has a rather large dog, the 'Frank' of the title. He wonders how many 'Franks' would fit in his bedroom, how long it would take to flood the bathroom if he left the tap on, how many humpback whales would fit in his house and so on. With humour and colourful well-drawn illustrations mathematical ideas such as number, volume and capacity are introduced. This book has stayed in print for twelve years for good reason: it is imaginative, funny and an excellent introduction to mathematical problem solving. Children can devise their own answers and strategies for solving the problems posed in the story. For instance, if the toaster sends the toast one metre into the air, how far would it send a piece of toast if it were as big as the house? Children can discuss their answers in small groups and illustrate their results with simple graphs.

Conclusion

The mathematics in picture storybooks can be explicit, implicit, or both. Often mathematics is not the author's intention but the ideas cannot be avoided because mathematics is so much a part of our existence. In the classroom, children's literature can be used as a springboard to mathematical activity (Reeves, 1984).

Illustrations are important in mathematical fiction, as are good characterisation and an element of humour — but no more so than in any other picture book. Teachers who have not used mathematical fiction to help teach mathematics before could start with some of the suggestions presented here and then look out for other suitable books. So, search for the story or illustrated book that is written by someone with a mathematical imagination, someone who is obviously excited by mathematics generally or simply the particular mathematical concept being portrayed. Some author/illustrators like Pamela Allen, Rod Clement and Matsumasa Anno just stand out from the pack!

References

Allen, P. (1986). *Mr Archimedes' Bath*. London: Hamish Hamilton.

Allen, P. (1988). *Who Sank the Boat?* Ringwood: Penguin Books Australia.

Anderson, J., Anderson, A. & Shapiro, J. (2000). *Parents using Picture Books to Support Mathematical Thinking*. Retrieved 5 August 2003 from http://www.nku.edu/~sheffield/andersonpbyd.html.

Anno, M. (1985). *Anno's Counting Book*. London: Macmillan Children's Books.

Anno, M. (Briggs, R. et al., illusts) (1986). *All in a Day*. London: Hamish Hamilton.

Anno, M. (1989). *Anno's Math Games 2*. New York: Philomel Books.

Anno, M. (1995). *Anno's Magic Seeds*. New York: Philomel Books.

Anno, M. & Anno, M. (1999). *Anno's Mysterious Multiplying Jar*. New York: Penguin.

Berenstain, J. & Berenstain, S. (1971). *Bears in the Night*. New York: Random House.

Branley, F. (2002). *Mission to Mars*. New York: Harper Collins.

Carle, E. (1970). *The Very Hungry Caterpillar*. London: Hamish Hamilton.

Carroll, L. (1999). *Alice's Adventures in Wonderland*. Sydney: Walker Books.

Clement, R. (1991). *Counting on Frank*. Sydney: Harper Collins.

Cullinan, B. E. (1971). *Literature for Children: Its discipline and content*. Dubuque, Iowa: W. C. Brown.

Donaldson, M. (1978). *Children's Minds*. New York: W. W. Norton.

Enzensberger, H. M. (1999). *The Number Devil: A mathematical adventure*. London: Granta Books.

Fearnley, J. (1999). *Mr Wolf's Pancakes*. London: Methuen Children's Books.

Garner, A. (1967). *The Owl Service*. Melbourne: Collins.

Golden, J. & Gerber, A. (1990). A semiotic perspective of text: The picture book story event. *Journal of Reading Behavior, 22*, 203–219.

Griffiths, R. & Clyne, M. (1991). Once upon a time… *The Australian Mathematics Teacher, 47* (1),10–13.

Guedj, D. (2000). *The Parrot's Theorem*. London: Weidenfeld & Nicolson.

Hutchins, P. (2001). *Rosie's Walk*. London: Random House.

Juster, N. (1989). *The Phantom Tollbooth*. Bath: Lythway Book/ Chivers Press.

Murphy, S. J. (2002). *Slugger's Car Wash*. New York: Harper Collins.

Nozaki, A. & Anno, M. (1985). *Anno's Hat Tricks*. New York: Harcourt Brace.

Padula, J. & Stacey, K. (1990). Learning basic concepts for early mathematics. *Australian Journal of Early Childhood, 15* (2), 34–7.

Reeves, N. (1984). Reading maths to children. In C. Bouffler (Ed.), *Australian Reading Association, 1*.

Scieska, J. & Smith, L. (1998). *Maths Curse*. Camberwell, Vic.: Puffin Books.

Seuss, Dr. (1965). *The 500 hats of Bartholomew Cubbins*. New York: Random House.

Smith, N. & Wendelin, K. (1981). Using children's books to teach mathematical concepts. *The Arithmetic Teacher*, November, 10–15.

Straine, L. (1969). Children's literature: An aid in mathematics instruction. *The Arithmetic Teacher*, October, 451–455.

Swift, J. (1995). *Gulliver's Travels*. New York: Gramercy Books/ Random House.

Tang, G. (2003). *Math-terpieces: The Art of problem solving*. New York: Scholastic Press.

Vivas, J. (1985). *The Grandma Poss Cookbook*. Adelaide: Omnibus Books/ Penguin.

Janice Padula is an experienced teacher who is currently working as a freelance author and editor. She is the author of numerous journal articles, mainly about mathematics education.